## 2-way bounding

example: $2x - 17 = -7$

$2x = 10$

$x = 5$

what if instead:

upper bound a) $x \leq 5$

lower bound b) $x \geq 5$

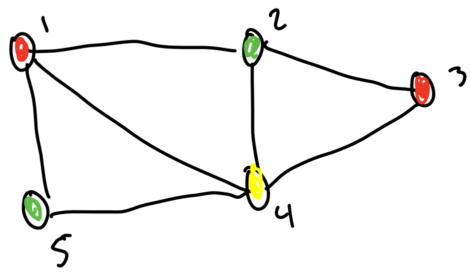
$\rightarrow x = 5$

if we establish both $x$ must = the bound.

## graph coloring

the coloring of a graph G assigns a color to each node in G such that no two adjacent nodes have same color.

the chromatic number of a graph G is the smallest number of colors needed to color G.




Given this graph, what is the chromatic number?

upper bound argument: chromatic number $\leq 3$ explicit coloring

lower bound argument: chromatic number $\geq 3$ * ① local feature ($K_3$)

② careful coloring

↓

the way you colored the graph is the only possible way.

* importantly, the upper and lower bound arguments have to be for the same chromatic number *

## Set equality

from Examlet A

Show $A=B$ by proving ① $A \subseteq B$ ② $B \subseteq A$.

ex) Let $A = \{15p + 9q \mid p, q \in \mathbb{Z}\}$
$B = \{\text{multiples of } 3\} \rightarrow \{3k \mid k \in \mathbb{Z}\}$

Show $A=B$.

First let's rewrite $B$ as $\{3k \mid k \in \mathbb{Z}\}$.

① We want to show $A \subseteq B$. Let $A, B$ be sets as defined above and let $x \in A$. Then, $x$ can be written as $x = 15p + 9q$ where $p, q \in \mathbb{Z}$. So, $x = 3(5p + 3q)$. Since $p, q \in \mathbb{Z}$, $5p + 3q \in \mathbb{Z}$. Let's call it $r \in \mathbb{Z}$. Then $x = 3r$, so $x \in B$.

② We want to show $B \subseteq A$. Let $A, B$ be sets as defined above and let $x \in B$. Then $x = 3k, k \in \mathbb{Z}$. We can write this as $x = -15k + 18k = 5(-3k) + 9(2k)$. $-3k \in \mathbb{Z}$, call it $a$. $2k \in \mathbb{Z}$, call it $b$. Then $x = 5a + 9b$ where $a, b \in \mathbb{Z}$, so $x \in A$.

Since $A \subseteq B$ and $B \subseteq A$, $A = B$.